Merkle Tree Ladder Mode: Reducing The Size Impact Of NIST PQC Signature Algorithms In Practice

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Video courtesy of RSA Conference

Introduction

NIST PQC Project has resulted in a remarkable variety of post-quantum algorithms

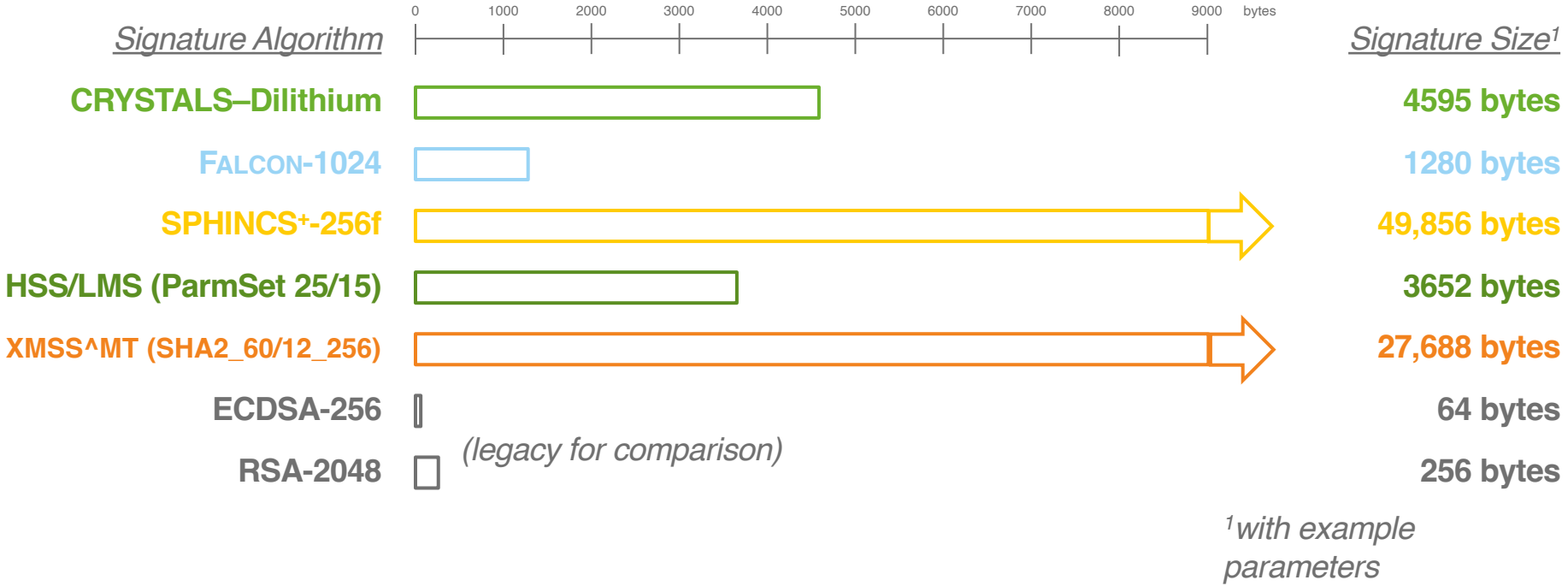
… but sizes of selected signature algorithms are large!

Large size may impact legacy applications with size constraints (e.g., DNSSEC)

How to reduce impact in practice?

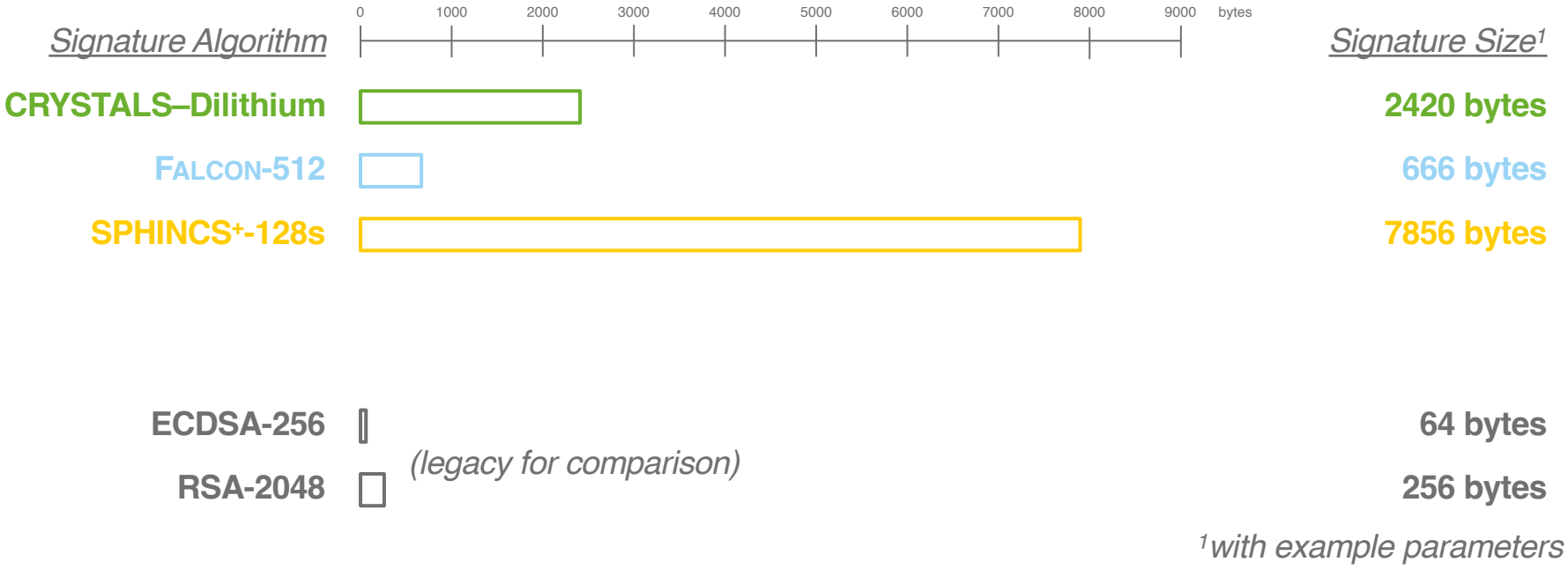
NIST PQC Signature Algorithms: Large Examples

Level V Parameters (256-bit security)



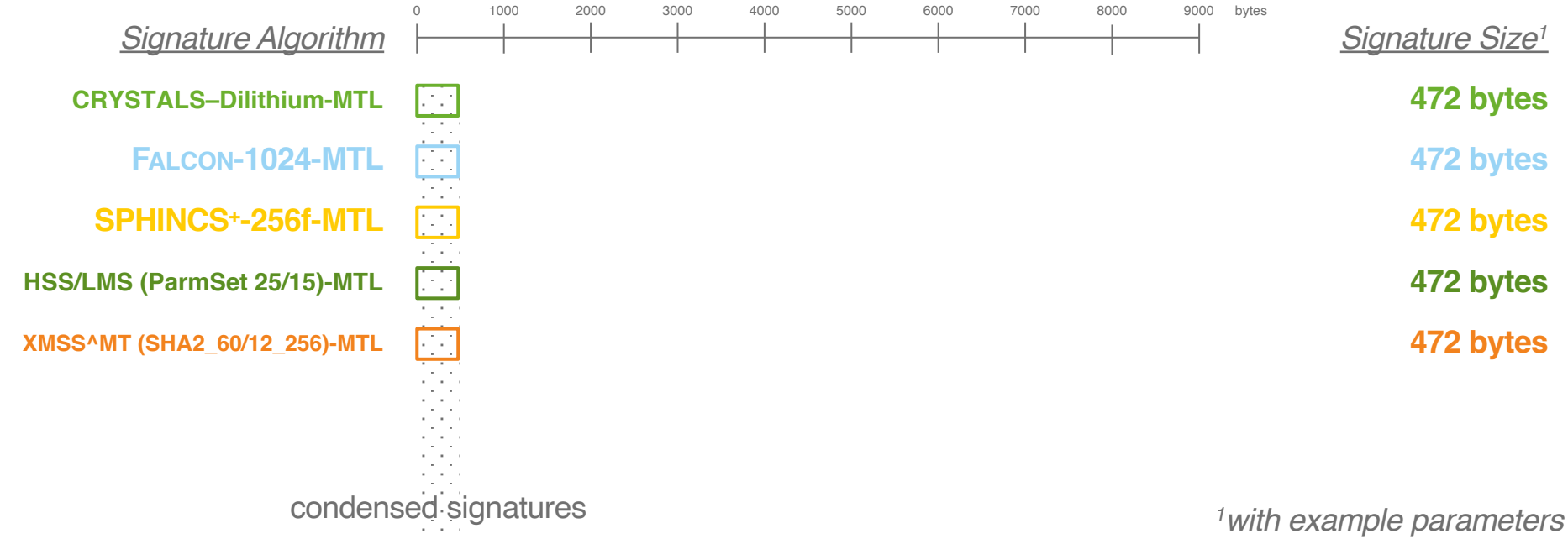
NIST PQC Signature Algorithms: Small Examples

Level I or II Parameters (128-bit+ security)



Merkle Tree Ladder Mode

Much Shorter… But How?

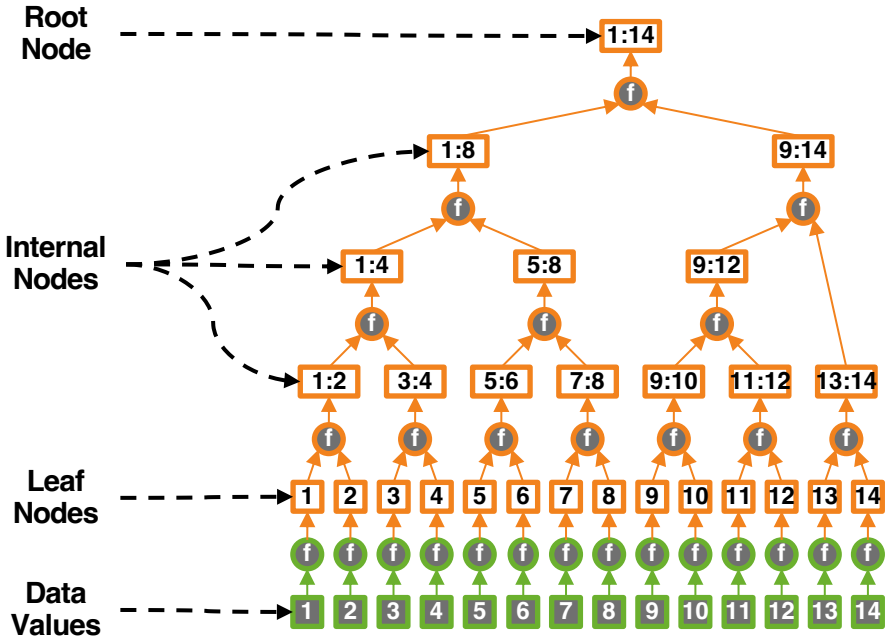


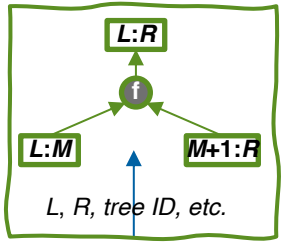
[1] *with example parameters*

Merkle Tree Ladders

Merkle Tree

Root Node Recursively Authenticates Data Values

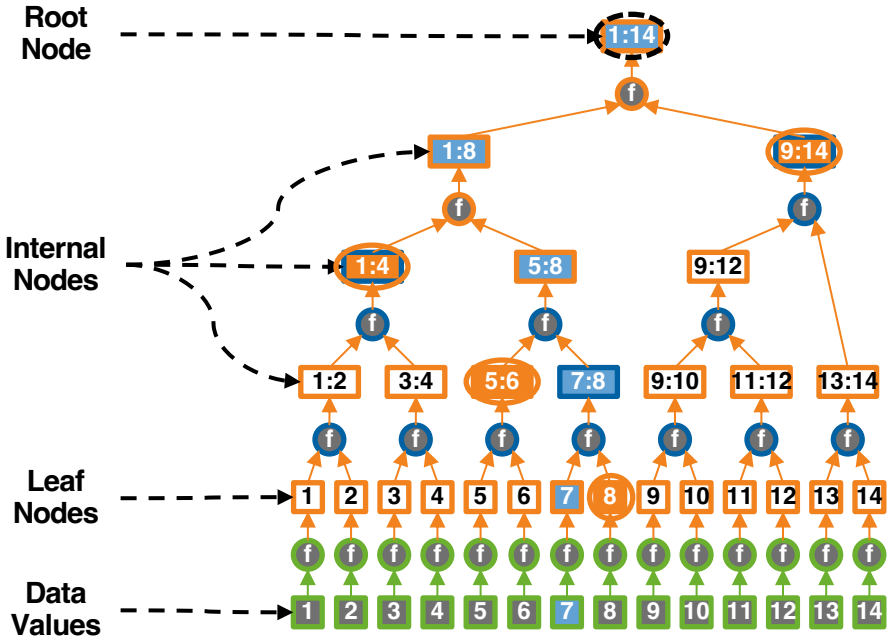




- *Merkle (1979)*
- *Not limited to power of 2*

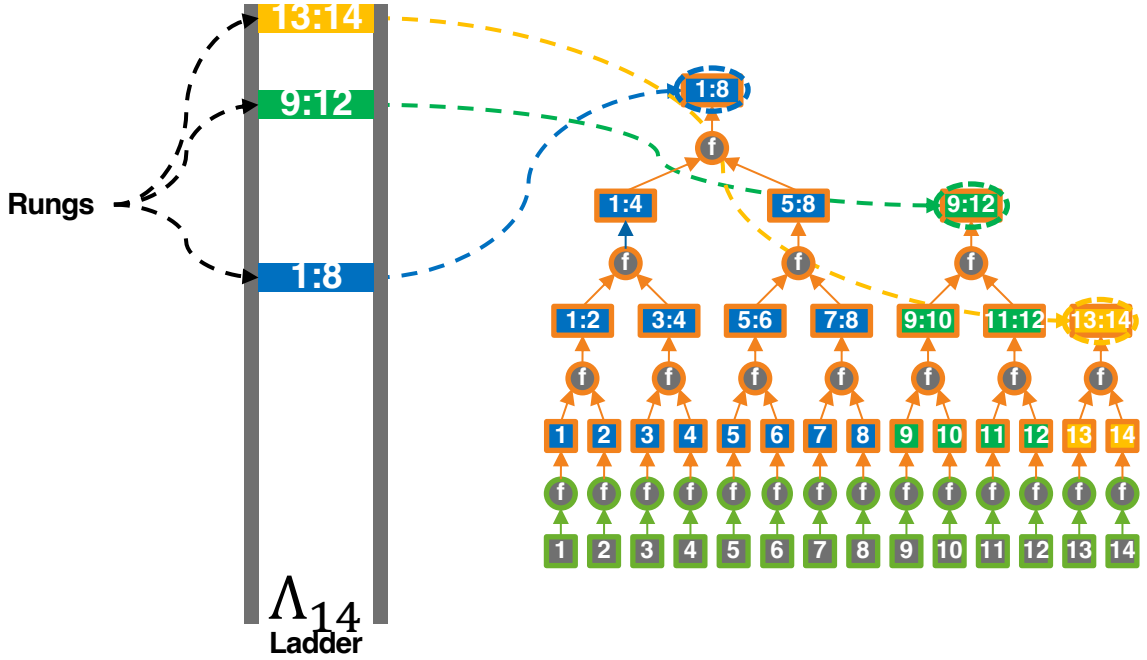
Authentication Path

Verify Data Value by Re-Hashing with Sibling Nodes



Merkle Tree Ladder

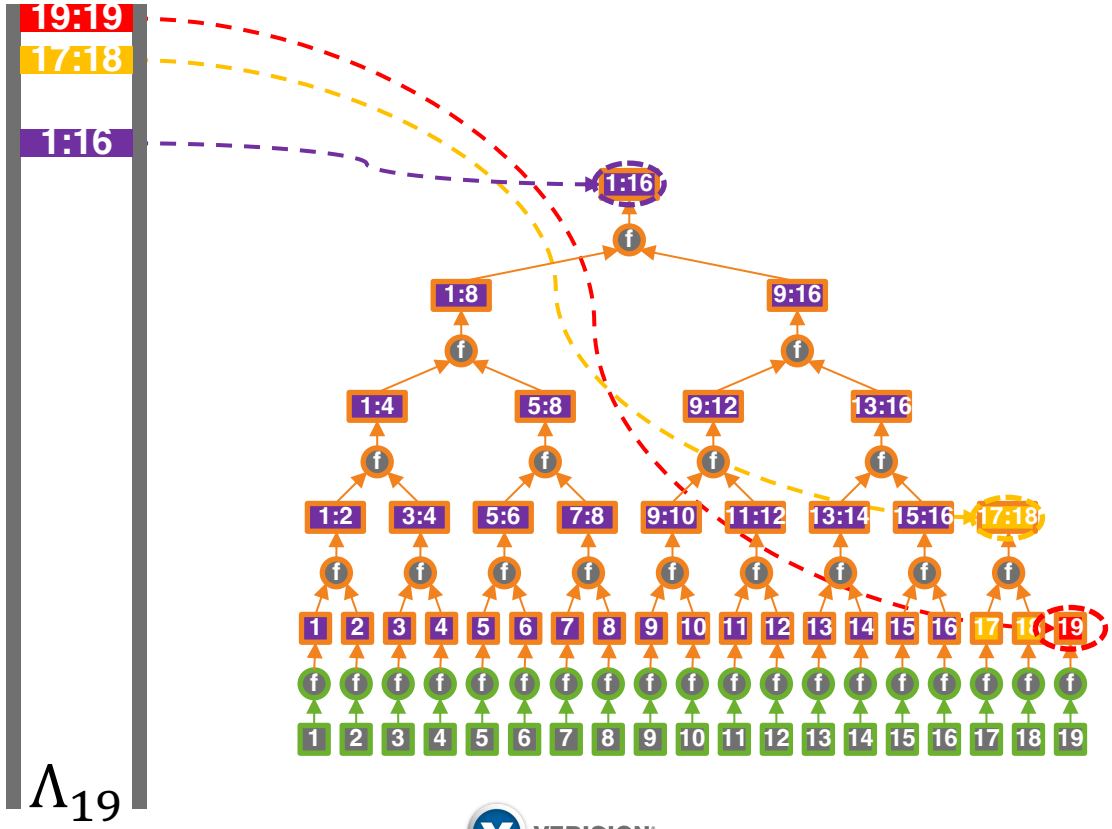
"Rungs" Collectively Authenticate All Data Values



- *"Binary" rung strategy shown; others possible*
- *Also called "binary numeral tree" (Champine 2021); "history trees" (Crosby-Wallach 2009), "Merkle Mountain Ranges" (Todd 2012)*

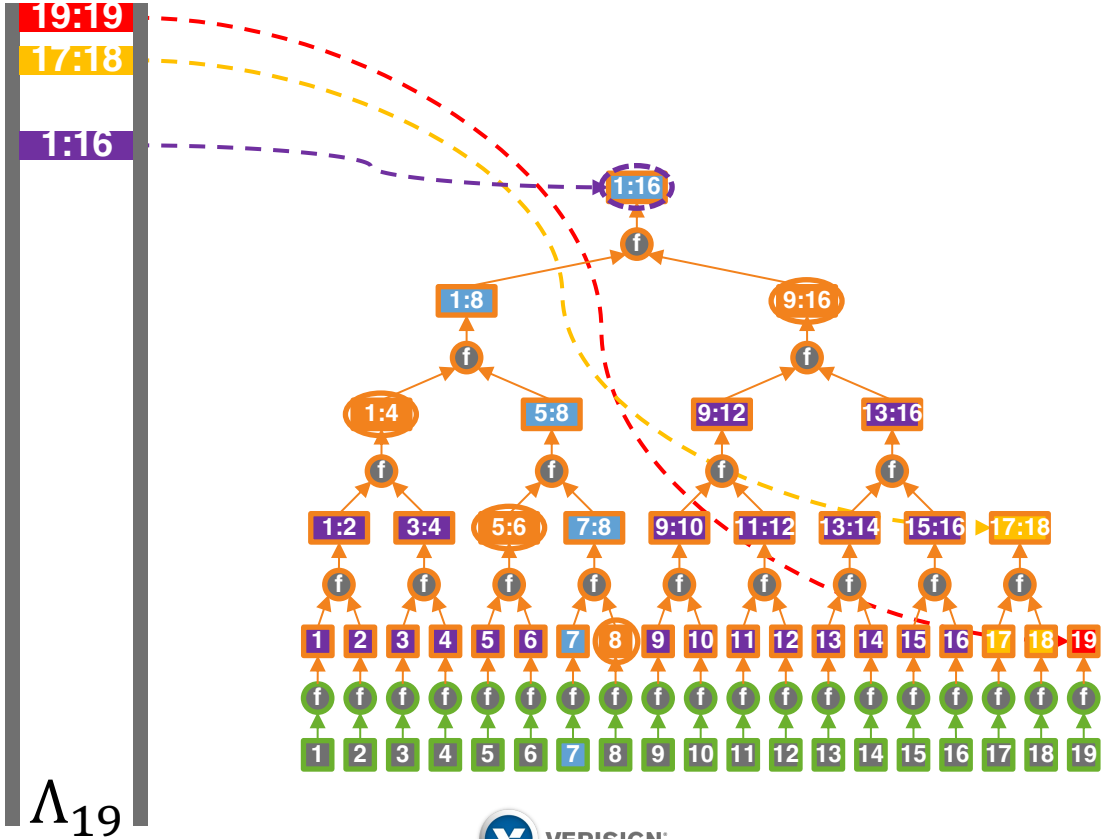
Ladder Evolution

Rungs Updated As New Data Values are Added



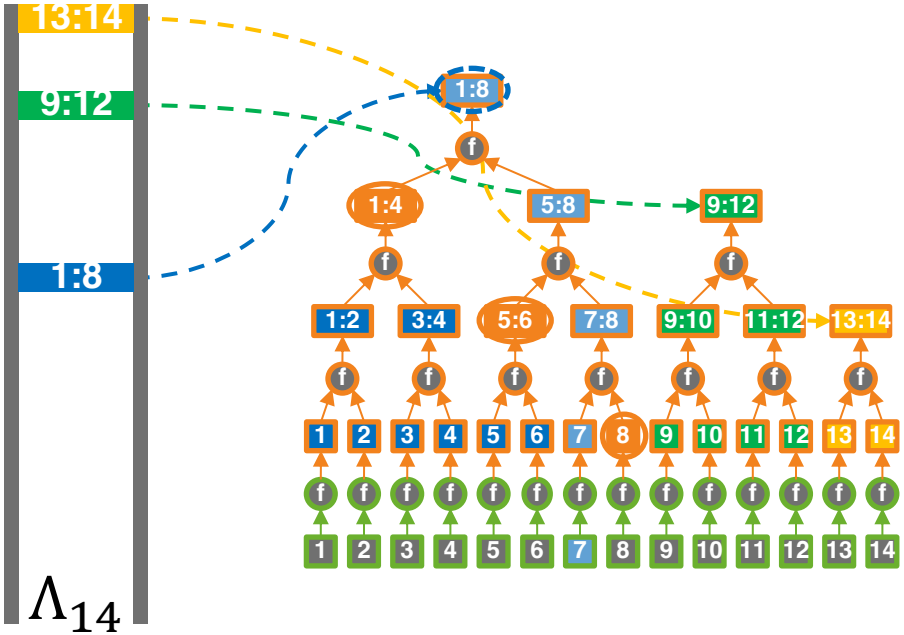
Authentication Path to Ladder

Verify Data Value by Re-Hashing with Sibling Nodes



Backward Compatibility

Authentication Path Can Be Verified Using Old Ladder

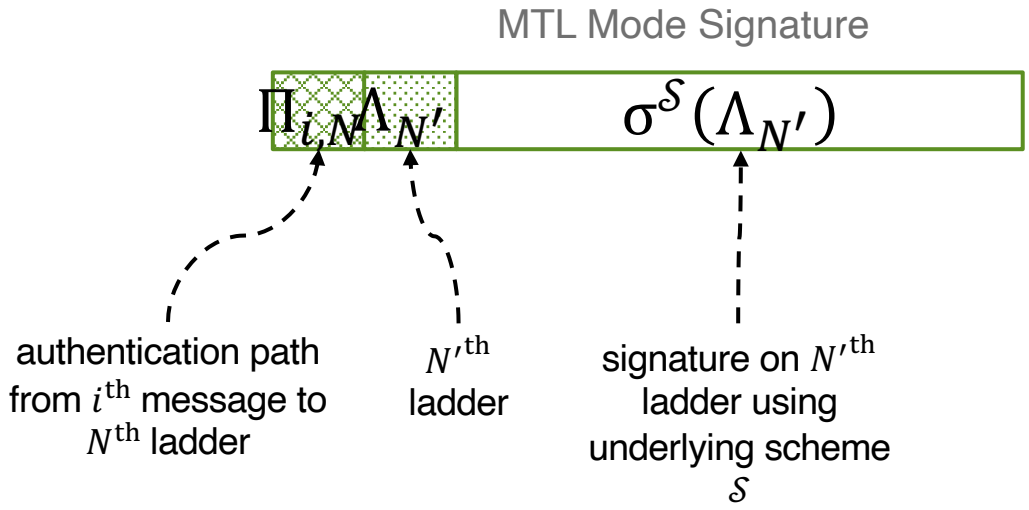


Analogous to "old-accumulator compatibility" (Reyzin-Yakoubov 2016)

Merkle Tree Ladder Mode

MTL Mode of Operation

Authentication Path + Ladder + Signature on Ladder



Signature Generation
produces initial signatures with
$i = N' = N$

Signature Verification
accepts signatures with
$i \leq N' \leq N$
verifiable via backward compatibility
(MTL mode is malleable)

MTL Mode: Evolve Trees, Sign Ladders

$\mathcal{S}$- MTL : *Stateful Transformation of Scheme $\mathcal{S}$*

Key Pair Generation

1. Generate $\mathcal{S}$ key pair
2. Add random series identifier to key pair
3. Add empty tree to private key

Signature Generation

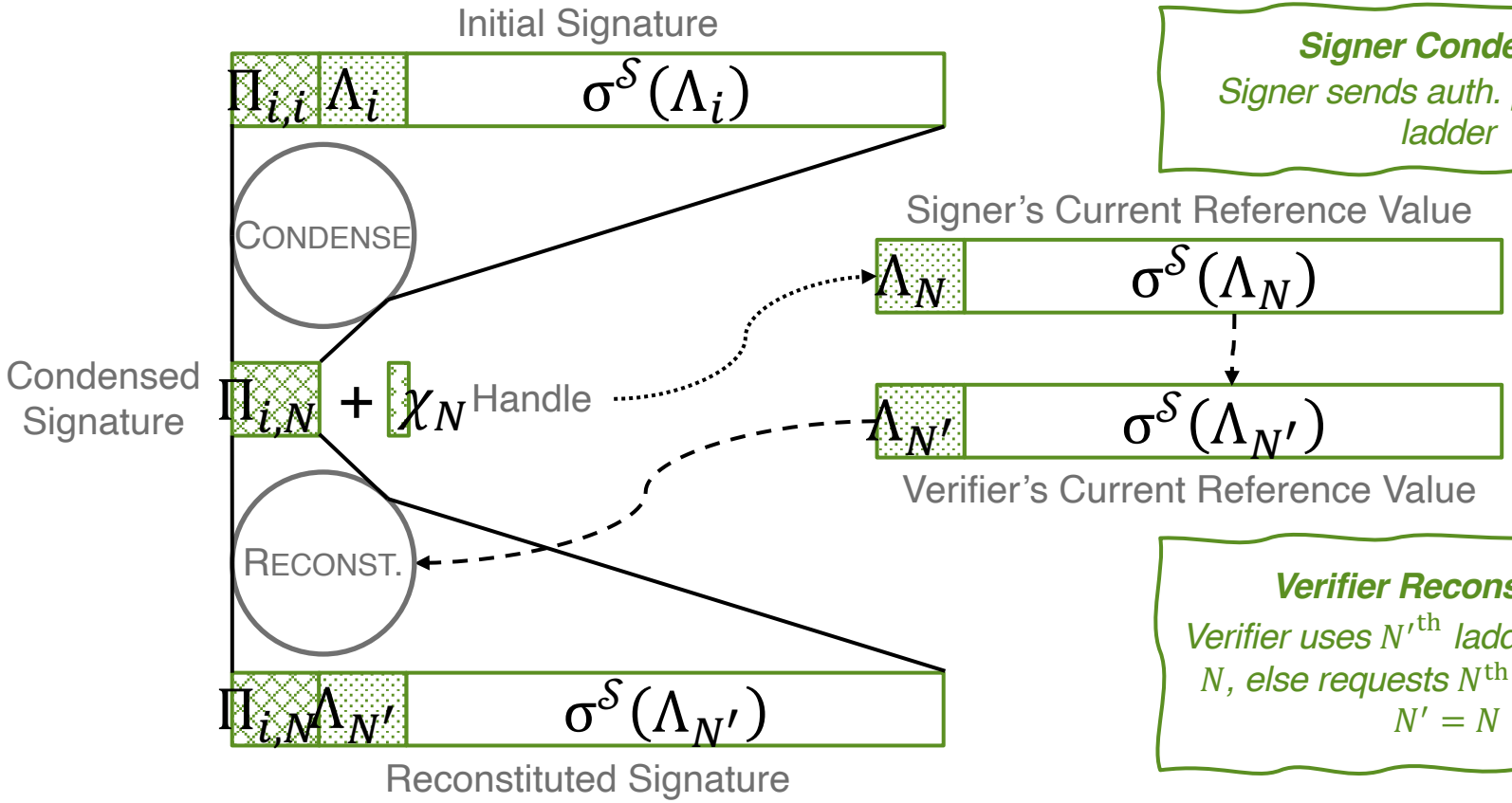
1. Hash message with randomizer → data value
2. Add data value to tree → new ladder
3. Sign ladder with $\mathcal{S}$
4. Form signature from randomizer + auth. path to ladder + ladder + $\mathcal{S}$ sig.

Signature Verification

1. Parse signature
2. Verify $\mathcal{S}$ sig. on ladder
3. Verify auth. path to ladder
4. Verify randomized hash on message

Multi-Target Design
All hash inputs include series identifier + message / node index

Condensing and Reconstituting MTL Mode Sigs.



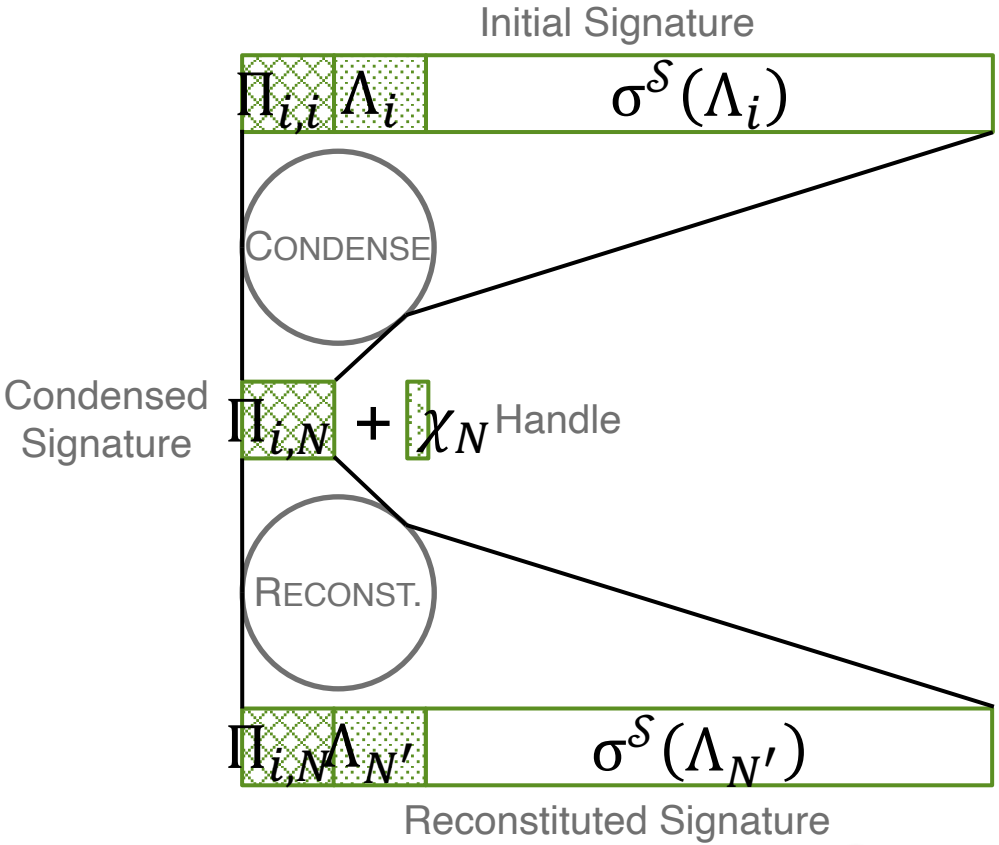
Signer Condenses

Signer sends auth. path to N^{th} ladder

Verifier Reconstitutes

Verifier uses N'^{th} ladder if $i \leq N' \leq N$, else requests N^{th} ladder, sets $N' = N$

Condensing and Reconstituting MTL Mode Sigs.



Anyone can condense *given access to signatures*

Condensation & reconstitution only involve public information → *don't impact security of MTL mode*

Anyone can reconstitute *given access to reference values*

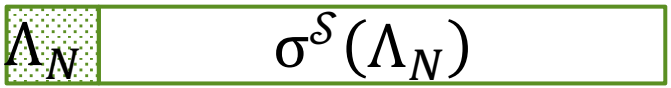
MTL Mode Signatures: Example Sizes (in bytes)

$N \approx 10{,}000$, *256-Bit Hash Output, 32-Bit Indexes*

Full Signature

Condensed Signature

Reference Value



- Ladders ≤ 14 hash values
- Authentication paths ≤ 13 hash values
- Full signature (also incl. series identifier, randomizer, i, N, N' and data value):

$$13{\times}32 + 14{\times}32 + 16 + 32 + 4 + 4 + 4 + 32 = 956 + \text{size of } \mathcal{S} \text{ sig.}$$

- Condensed signature (also incl. series identifier, randomizer, i, N'):

$$13{\times}32 + 16 + 32 + 4 + 4 = 472$$

- Reference value (also incl. N):

$$14{\times}32 + 4 = 452 + \text{size of } \mathcal{S} \text{ sig.}$$

Security Proofs

Security Against Existential Forgery

Proof Sketch

Forgery Cases

- $\mathcal{S}$ forgery
- auth. path second preimage

  (multi-target / multi-function)
- randomized hash second preimage

  (nonce extended-target)

Signature Verification

1. Parse signature
2. Verify $\mathcal{S}$ sig. on ladder
3. Verify auth. path to ladder
4. Verify randomized hash on message

- *Follows XMSS-T definitions (Hülsing et al. 2016)*

Multi-Target Design

All hash inputs include series identifier + message / node index

Random Oracle Model Bounds

Against Classical Adversaries

$$\mathrm{InSec}^{\text{EU-CMA}}(\mathcal{S}\text{-MTL}) \leq \mathrm{InSec}^{\text{EU-CMA}}(\mathcal{S}) + \frac{(q+1)}{2^{\ell}} + \frac{q}{2^{\ell_c}}$$

where

- q is number of hash queries
- ℓ is size of hash output in bits
- ℓ_c is size of randomizer in bits
- randomized hash & hash function families for tree leaves & internal nodes are modeled as random oracles

Motivated by security proofs for LMS (Katz 2016; Fluhrer 2017)

MTL Mode: Evolve Trees, Sign Ladders

$\mathcal{S}$-MTL : *Stateful Transformation of Scheme $\mathcal{S}$*

Hash Function Instantiation

- "program" pseudorandom function oracle to embed second preimage

Signature Generation

1. Hash message with randomizer → data value
2. Add data value to tree → new ladder
3. Sign ladder with $\mathcal{S}$
4. Form signature from randomizer + auth. path to ladder + ladder + $\mathcal{S}$ sig.

Motivated by XMSS-T (Hülsing et al. 2016) and SPHINCS⁺ (2022) designs

Mostly Standard Model Bounds

Against Classical Adversaries

$$\mathrm{InSec}^{\text{EU-CMA}}(\mathcal{S}\text{-MTLr}) \leq$$
$$\mathrm{InSec}^{\text{EU-CMA}}(\mathcal{S}) + \mathrm{InSec}^{\text{MM-SPR}}(H'_{\text{leaf}}) +$$
$$\mathrm{InSec}^{\text{MM-SPR}}(H'_{\text{int}}) + \frac{(q+1)}{2^{\ell}} + \frac{q}{2^{\ell_c}}$$

where

- q is number of hash queries
- ℓ is size of hash output in bits
- ℓ_c is size of randomizer in bits
- H'_{leaf}, H'_{int} are hash function families for tree leaves & internal nodes
- randomized hash & pseudorandom functions are modeled as random oracles

Mostly Standard Model Bounds
Against Quantum Adversaries

$$\mathrm{InSec}^{\text{EU-CMA}}(\mathcal{S}\text{-}\,\mathrm{MTLr}) \leq$$
$$\mathrm{InSec}^{\text{EU-CMA}}(\mathcal{S}) + \mathrm{InSec}^{\text{MM-SPR}}(H'_{\text{leaf}}) +$$
$$\mathrm{InSec}^{\text{MM-SPR}}(H'_{\text{int}}) + \frac{8(q+q_s+2)^2}{2^{\ell}} + 3q_s\sqrt{\frac{q+q_s+1}{2^{\ell_c}}}$$

where terminology is as above plus

- q_s is number of signature queries

Based on (Grilo et al. 2021) improvement to (Bos et al. 2020) for hash-and-sign with nonce

Bit Security of MTL Mode

Incremental to Underlying Signature Scheme

Size of Hash Output ℓ	Size of Randomizer ℓ_c	Number of Signatures q_s	Classical Security (bits)	Quantum Security (bits)
256	256	2^{20}	253	125
		2^{60}		
	259	2^{64}		
128	128	2^{20}	125	61
		2^{60}		

Double-Signing Impact in MTL Mode

- **Chosen State Attacks:** Suppose adversary can control MTL mode state, get signatures on multiple messages with same index i
- If $\mathcal{S}$ is LMS, XMSS: one-time private key material revealed → security *compromised*
- If $\mathcal{S}$ is Dilithium, FALCON, SPHINCS$^+$: multiple hash targets with same index → security gradually *degraded*
- **Reimagine the State:** MTL mode uses state to bring benefits (condensability, etc.) without the usual risk

Practical Impact

Ladder Endurance: General Model

How Many Signatures Until Next Reference Value?

N_0 initial messages

α new messages signed per “iteration”

ρ messages randomly selected to verify per iteration — with current condensed sig. $\Pi_{i,N}$

Verifier initially has reference value with ladder Λ_{N_0}

How many messages until verifier needs new reference value, i.e., $i > N_0$?

Ladder Endurance: General Model, cont'd

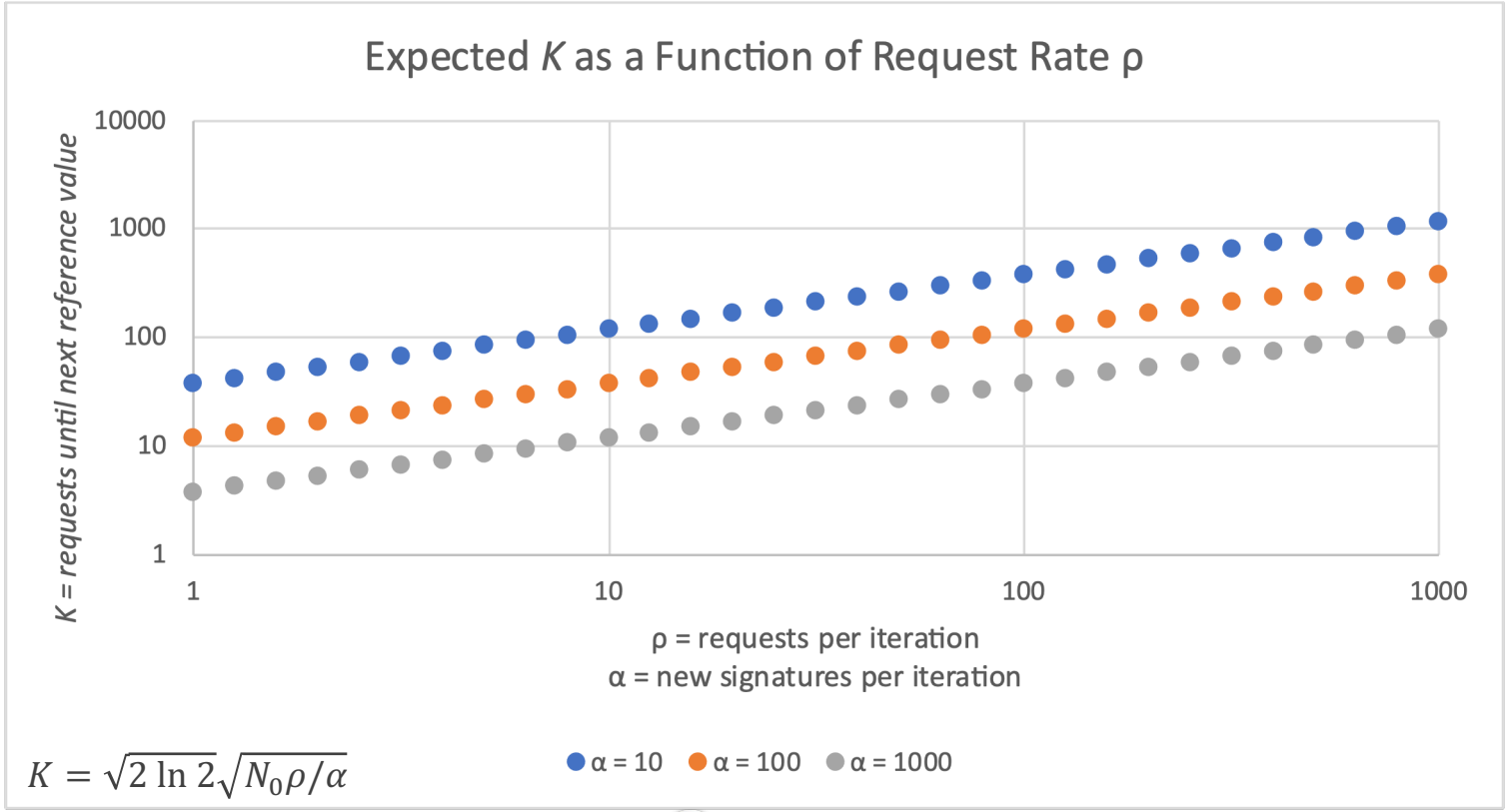
How Many Signatures Until Next Reference Value?

- $\text{Prob}[\kappa \text{ "good" iterations}] \approx \prod_{t=1}^{\kappa} \left(\frac{N_0}{N_0+t\alpha}\right)^{\rho} \approx \exp\left(-\frac{\kappa^2 \alpha\rho}{2N_0}\right)$

- Probability $\approx 1/2$ when $\kappa \approx \sqrt{2 \ln 2}\sqrt{N_0/\alpha\rho}$
- Endurance: $\sqrt{2 \ln 2}\sqrt{N_0/\alpha\rho}$ iterations → $K = \sqrt{2 \ln 2}\sqrt{N_0\rho/\alpha}$ messages

Expected Ladder Endurance

w/Request Rate ρ, Update Rate α; Assume $N_0 = 10{,}000$



$$K = \sqrt{2 \ln 2}\sqrt{N_0 \rho / \alpha}$$

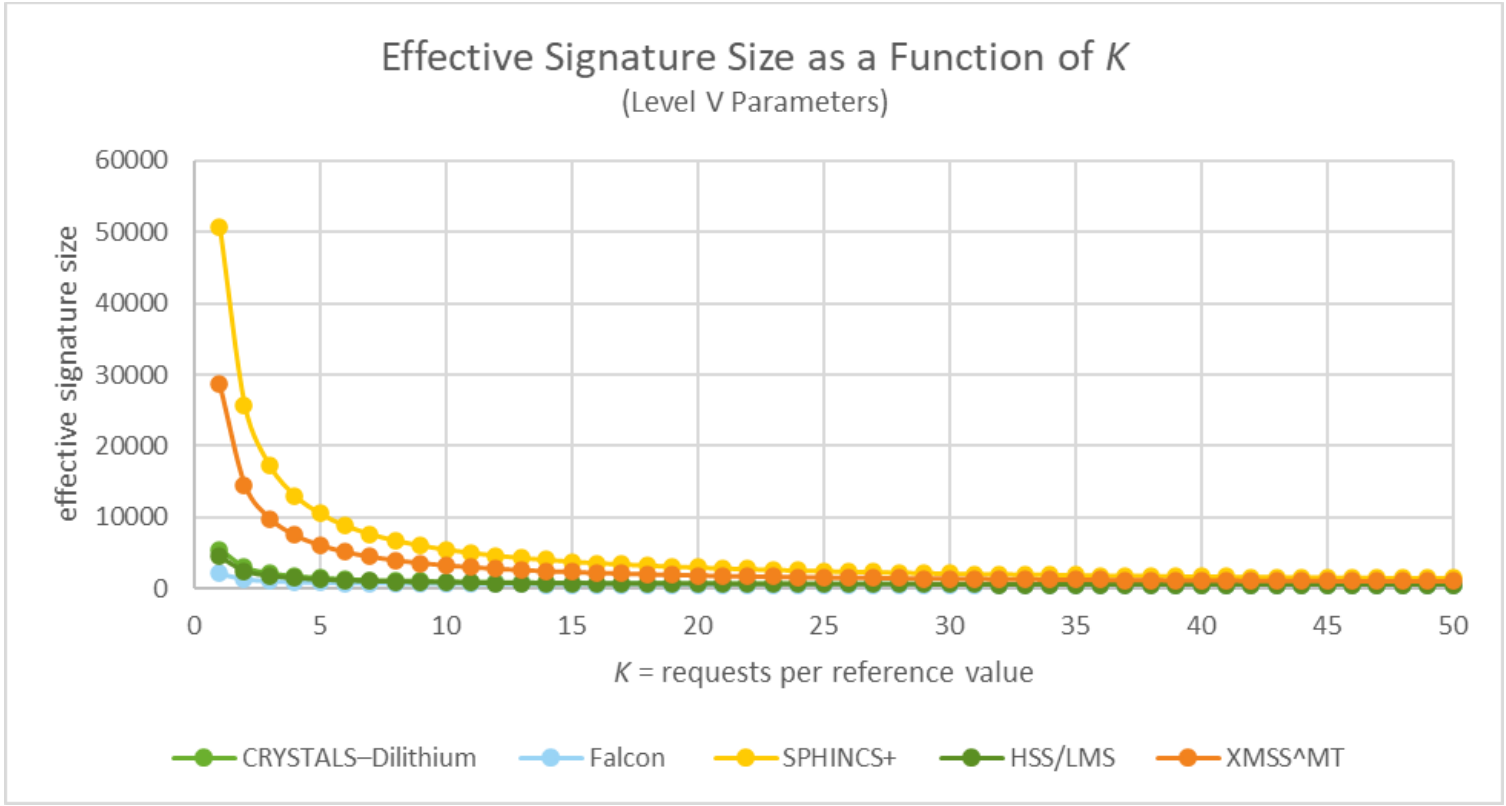
Effective Signature Size for Large Examples

MTL Mode Parameterized for ~10,000-Message Series

Signature Algorithm (NIST Level V)	Underlying Signature Size (bytes)	Condensed MTL Mode Signature Size $\lvert\varsigma\rvert$	Reference Value Size $\lvert\upsilon\rvert$	Effective MTL Mode Signature Size ($K = 10$)
CRYSTALS-Dilithium	4595	472	5047	976.7
FALCON-1024-MTL	1280	472	1732	645.2
SPHINCS⁺-256f-MTL	49,856	472	50,308	5502.8
HSS/LMS (ParmSet 25/15)-MTL	3652	472	4104	882.4
XMSS^MT (SHA2_60/12_256)-MTL	27,688	472	28,140	3286

Effective Signature Size

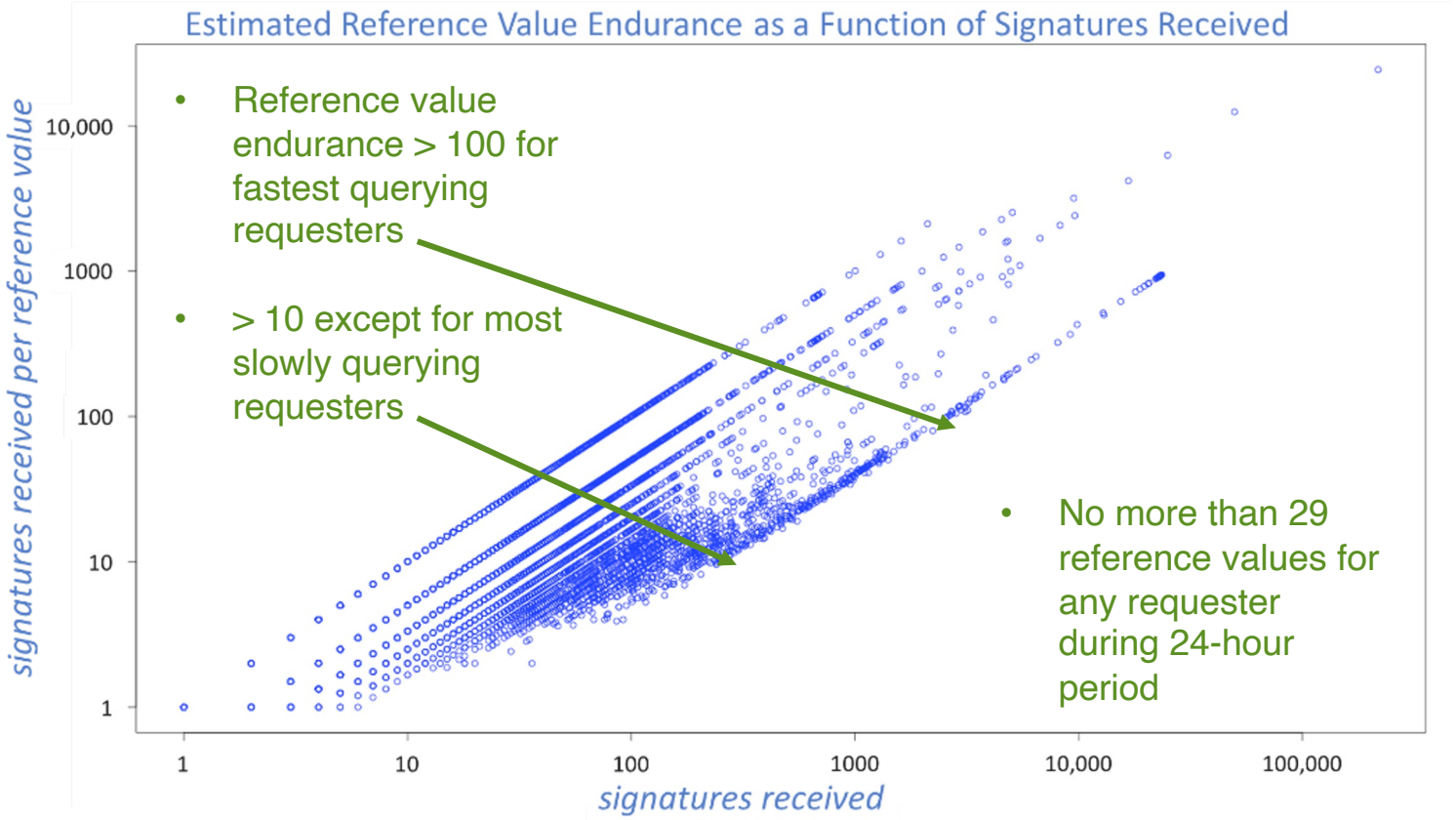
Converging to Condensed Signature Size as K Increases



DNSSEC Use Case

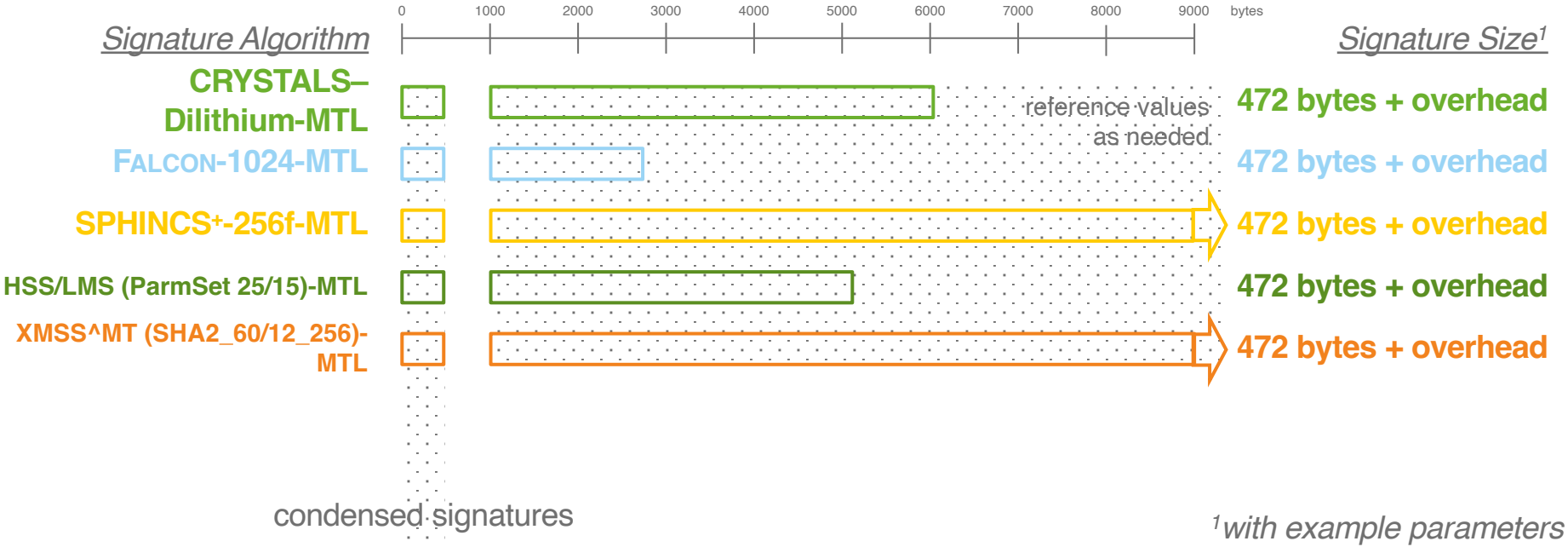
- DNSSEC signatures authenticate DNS records
  - Typically precomputed by signers, at top levels of DNS
  - Requested by verifiers based on interest (~ our model)
- We analyzed one of DNS-OARC's Day In The Life (DITL) data sets from 2015 to see what would have happened if MTL mode had been applied
- Measurement: How often does DNSSEC "signature inception" reach new maximum for a given verifier?
  - Proxy for requesting new reference values

DNSSEC Results



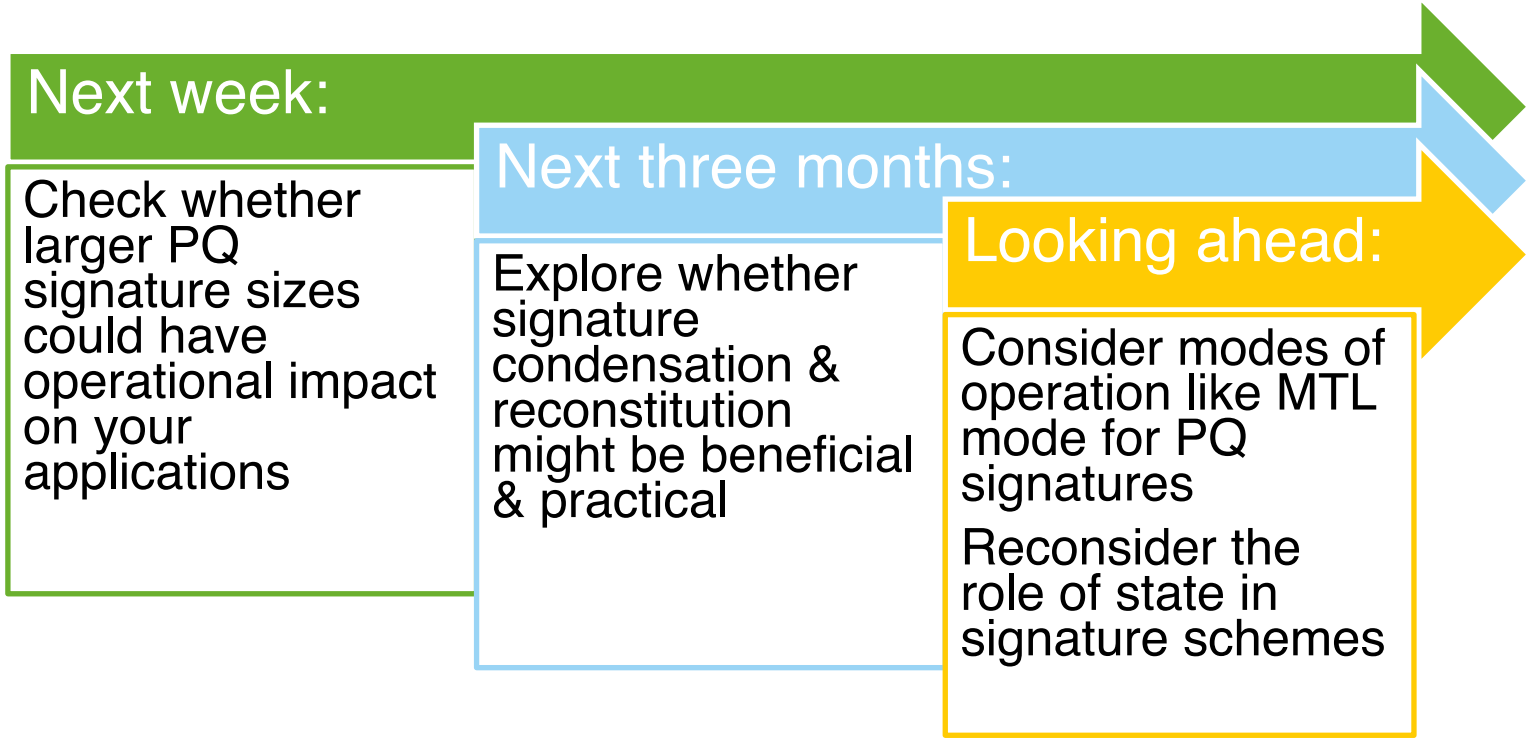
Summary: Reducing Effective Size Impact

Send Condensed Signatures, Look Up Reference Values



[1] *with example parameters*

Applying MTL Mode



Conclusion

Merkle Tree Ladder mode transforms any signature scheme into a stateful, *condensable* signature scheme

Condensation can reduce size impact in message-series-signing scenarios, e.g., DNSSEC

Security bounds are comparable to those for other hash-based signature schemes

Reimagine the state: MTL mode uses state to bring benefits without the usual risk

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